

Does “Doppler boosting / de-boosting” exist within the framework of General Relativity? First approximation.

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Abstract

The “Doppler boosting / de-boosting” relativistic effect increases / decreases the apparent luminosity of approaching / receding sources of radiation. This effect was analyzed in detail within the Special Relativity framework and was confirmed in many astronomical observations. It is however not clear if “Doppler boosting / de-boosting” exists in the framework of General Relativity as well, and if it exists, which equations describe it. Presented thought experiment, using “Einstein’s elevator” model and Einstein’s “Equivalence principle” allows to obtain the formula for “Doppler boosting / de-boosting” for a uniform gravitational field within the vicinity of the emitter/receiver.

Key words: Doppler boosting, Equivalence principle, de-boosting, General Relativity

1 “Doppler boosting / de-boosting” in Special Relativity

The “Doppler boosting / de-boosting” relativistic effect increases / decreases the apparent luminosity of approaching / receding sources of radiation.

The illustration fig.1 (re-drawn from the figure 11.2 of [Granot & Ramirez-Ruiz, 2012](#)) illustrates the distribution of energy in “boosting” and “de-boosting” zones of relativistic radiation sources. The arrows show the directions of photons in the observer’s frame for a source that emits isotropically in its own rest frame and moves to the right at various velocities. In accordance to ([Granot & Ramirez-Ruiz, 2012](#)) “for a relativistic source moving with a Lorentz factor $\Gamma = (1 - \beta^2)^{-1/2} \gg 1$ (in the lab frame), half of the photons and most (3/4) of the emitted energy are within an angle of $1/\Gamma$ around its direction of motion”.

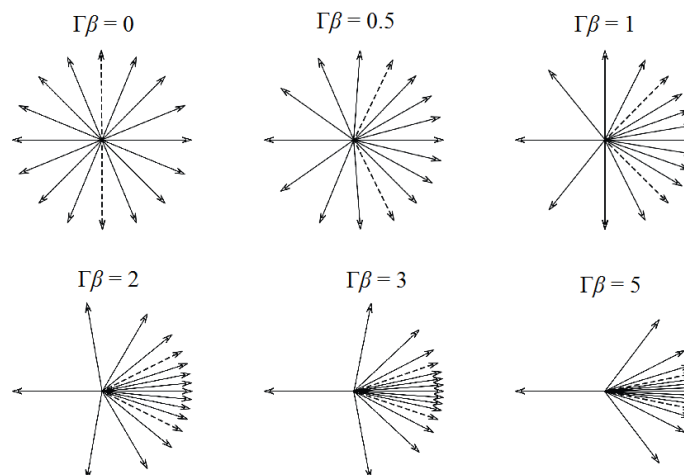


Figure 1. Relativistic boosting and de-boosting. Half of the photons (and 3/4 of the radiated energy) are within an angle of $1/\Gamma$ around the direction of motion (between the dashed arrows, which correspond to $\theta=90^\circ$ in rest frame.) Re-drawn from the figure 11.2 of [Granot & Ramirez-Ruiz, 2012](#).

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This effect was analyzed in detail within the Special Relativity framework and was confirmed in many astronomical observations. “Doppler boosting” and “de-boosting” were successfully taken into account in the analysis of relativistic jets of active galactic nuclei (AGN), in the observation of double white dwarfs, in research of pulsars, in search of exoplanets and stars in binary systems, and in the analysis of gamma-ray bursts (GRBs) ([Kellermann, Kovalev & Lister 2007](#); [Lister 2003](#); [Shporer et al., 2010](#); [Zhou & Su 2006](#); [Yang 2010](#); [Li K. L. et al, 2018](#); [Placek 2019](#); [Massi & Torricelli-Ciamponi 2014](#)).

The relationship between the apparent luminosities (L), intrinsic luminosities (L_o) and spectral index given by formula (1) ([Lister 2003](#))

$$L = L_o \delta^p \quad (1).$$

Here the Doppler factor δ is

$$\delta = \gamma^{-1} (1 - \beta \cos \theta)^{-1} \quad (2),$$

the Lorentz factor γ is

$$\gamma = (1 - \beta^2)^{-1/2} \quad (3),$$

the velocity β is the velocity v of a relativistic light source normalized to the speed of light c

$$\beta = v/c \quad (4),$$

θ is the angle between line of sight and the velocity direction, α is the spectral index ($S_\nu \propto \nu^\alpha$) of emitter, and $p=3-\alpha$ for a discrete emitting region.

2 “Doppler boosting / de-boosting” in General Relativity

It is however not clear if “Doppler boosting / de-boosting” also exists in the framework of General Relativity, and if it exists, which equations describe it. The following section will analyze this in further detail.

“Doppler boosting” is a combination of 3 individual relativistic effects, namely relativistic aberration, time dilation and Doppler shifts. All 3 of these effects also exist within General Relativity. Gravitational aberration has been known since the famous experiment that detected gravitational deflection in 1919 ([Dyson, Eddington & Davidson, 1919](#)) to the modern observation of gravitational lensing and black hole shadows ([Chang & Zhu, 2020](#); [Arakida, 2021](#)). Gravitational time dilation was confirmed since the ([Pound & Rebka, 1959](#)) experiment and has continued with recent research of gravitational redshift tracked by “Radioastron” ([Nunes et al., 2020](#)). Doppler shifts are the basis of cosmology.

As per the thought experiment, described in “Einstein’s elevator” model ([Misner, Thorne & Wheeler, 1973](#)), we can expect that the observer in the elevator that moves with a constant acceleration $g>0$ will observe “Doppler de-boosting” from the radiation emitter, which is located in the same elevator, below the receiver. ([Misner, Thorne & Wheeler, 1973](#) used word “rocket” instead of “elevator”, but we will continue to use elevator here as per Einstein’s term.)

Indeed, in the thought experiment ([Misner, Thorne & Wheeler, 1973](#)) the receiver, which is located within the elevator above the emitter, observes Doppler’s redshift as the velocity of the

receiver is greater than the velocity of the emitter (located within the same elevator) as soon as the acceleration $g > 0$. This difference in velocities between the emitter and receiver occurs due to the fact that during the time interval ΔT required for photons radiated by the emitter to reach the receiver, the receiver's velocity increases due to the elevator's acceleration.

For the same reason, we can expect the luminosity of the emitter measured by the receiver to appear reduced, since the velocity of the receiver is greater than the velocity of the emitter (due to acceleration) and the luminosity of the receding emitter will be de-boosted.

Since the observer within the elevator does not know if the acceleration g is produced by gravity or by the elevator's acceleration, in accordance with the Einstein's "Equivalence principle", we can expect that "Doppler de-boosting" will be observed for gravity as well.

In the first approximation we may try to calculate the gravitational boosting/de-boosting effect using the relationship between the apparent luminosities (L), intrinsic luminosities (L_o) and spectral index given by formula (1) above.

In the model with "Einstein's elevator" the velocity v of the receiver is $v = g\Delta T$, where ΔT is the time required by photons radiated by the emitter to reach the receiver and g is the acceleration.

If h is the distance between the emitter and receiver, then for a short h the time interval ΔT is $\Delta T \approx h/c$. (The distance h is "short" if photon's additional travel due to acceleration of receiver is negligible compared to h .)

$$\text{Therefore, velocity } v \text{ of the receiver is } v = g\Delta T = gh/c \quad (5),$$

$$\text{and normalized velocity } \beta = v/c = gh/c^2 \quad (6).$$

$$\text{Correspondingly } \gamma = (1 - \beta^2)^{-1/2} = [1 - (gh/c^2)^2]^{-1/2} \quad (7).$$

$$\text{Since } \delta = \gamma^{-1} (1 - \beta \cos \theta)^{-1}, \text{ we have } \delta = [1 - (gh/c^2)^2]^{1/2} (1 - gh/c^2 \cos \theta)^{-1} \quad (8).$$

Assuming that the receiver is located exactly above the emitter, in the direction of acceleration g (i.e. that $\theta = 180^\circ$), we have

$$\delta = [1 - (gh/c^2)^2]^{1/2} (1 + gh/c^2)^{-1} \quad (9),$$

and

$$L = L_o \{ [1 - (gh/c^2)^2]^{1/2} (1 + gh/c^2)^{-1} \}^p \quad (10).$$

Since $p = 3 - \alpha$ for a discrete emitting region we have

$$L = L_o \{ [1 - (gh/c^2)^2]^{1/2} (1 + gh/c^2)^{-1} \}^{(3-\alpha)} \quad (11),$$

where α is the spectral index ($S_\nu \propto \nu^{-\alpha}$) of the emitter.

Formula (11) was obtained in simplified conditions of a uniform gravitational field (since we used the model with "Einstein's elevator" and the Equivalence principle) and for short distances h between the emitter and receiver. This is the first step towards the complete set of equations

that describe the "Doppler boosting / de-boosting" effect within the General Relativity framework.

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